Engineering Applications of Artificial Intelligence 26 (2013) 430–445

Contents lists available at SciVerse ScienceDirect

Engineering Applications of Artificial Intelligence

journal homepage: www.elsevier.com/locate/engappai

Multi-objective optimization of two stage thermoelectric cooler using a modified teaching–learning-based optimization algorithm

R. Venkata Rao*, Vivek Patel

Department of Mechanical Engineering, S.V. National Institute of Technology, Ichchanath, Surat 395 007, Gujarat, India

A R T I C L E I N F O

Article history:
Received 21 August 2011
Received in revised form
23 November 2011
Accepted 21 February 2012
Available online 19 March 2012

Keywords:
Two stage thermoelectric cooler
Cooling capacity
Coefficient of performance
Thermoelectric modules
Teaching–learning-based optimization

A B S T R A C T

Teaching–learning-based optimization (TLBO) is a recently developed heuristic algorithm based on the natural phenomenon of teaching–learning process. In the present work, a modified version of the TLBO algorithm is introduced and applied for the multi-objective optimization of a two stage thermoelectric cooler (TEC). Two different arrangements of the thermoelectric cooler are considered for the optimization. Maximization of cooling capacity and coefficient of performance of the thermoelectric cooler are considered as the objective functions. An example is presented to demonstrate the effectiveness and accuracy of the proposed algorithm. The results of optimization obtained by using the modified TLBO are validated by comparing with those obtained by using the basic TLBO, genetic algorithm (GA), particle swarm optimization (PSO) and artificial bee colony (ABC) algorithms.

© 2012 Elsevier Ltd. All rights reserved.

1. Introduction

The application of thermoelectric coolers (TECs) has grown appreciably because of the need for a steady, low-temperature, environment friendly operating environment for various applications such as aerospace, military, medicine, biology and other electronic devices. However, the cooling capacity and coefficient of performance (COP) of TECs are low compared with traditional devices like vapor compression system, vapor absorption system, etc. Therefore, performance improvement of the TECs is an important issue in their applications (Cheng and Lin, 2005; Pan et al., 2007).

With the help of one stage TEC, maximum 70 K temperature difference is produced when its hot end is maintained at room temperature. So, when a large temperature difference is required, two stage TECs should be employed (Rowe, 1996). Usually two-stage TECs are commercially arranged in cascade; the cold stage is attached to the heat source and the hot stage pumps total heat to the environment. Moreover the two stage TECs are arranged in two different design configurations as shown in Fig. 1. In such two stage TECs, the determination of the number of thermoelectric (TE) modules in hot stage and cold stage as well as the supply current to the hot stage and the cold stage are important for improving the COP and cooling capacity of TECs. Moreover, the consideration of temperature dependent material properties and existence of thermal and electric contact resistance between the contact surfaces of TECs make the determination of these parameters complex (Xuan et al., 2002a; Cheng and Shih, 2006).

Traditional methods of optimization do not fare well over a broad spectrum of problem domains as they tend to obtain a local optimal solution. Considering the drawbacks of traditional optimization techniques, attempts can be made to optimize the TECs problem using nature inspired optimization algorithms like genetic algorithm (GA), particle swarm optimization (PSO), artificial bee colony (ABC), harmony search (HS), etc. Among all, most commonly used evolutionary optimization technique is the genetic algorithm. However, the effective working of GA requires determination of optimum controlling parameters such as crossover rate and mutation rate. Similarly, PSO requires determination of inertia weight, social and cognitive parameters. ABC requires determination of employed bees, onlooker bees, scout bees and limit. HS requires harmony memory consideration rate, pitch adjusting rate, and number of improvisations. In above mentioned algorithms, a change in the controlling parameters changes the effectiveness of the algorithm.

Recently, Rao et al. (2011) proposed a teaching–learning based optimization (TLBO) algorithm based on the natural phenomenon of teaching and learning. TLBO is an algorithm-specific parameter-less algorithm. The implementation of TLBO does not require the determination of any controlling parameters which makes the algorithm robust and powerful. In this work, some modifications to the standard TLBO algorithm are introduced and the performance of the modified TLBO algorithm is investigated for multi-objective optimization of a two stage TEC.

* Corresponding author. Tel.: +91 261 2201661; fax: +91 261 2201571.
E-mail address: ravipudirao@gmail.com (R. Venkata Rao).

0952-1976/$ - see front matter © 2012 Elsevier Ltd. All rights reserved.
doi:10.1016/j.engappai.2012.02.016

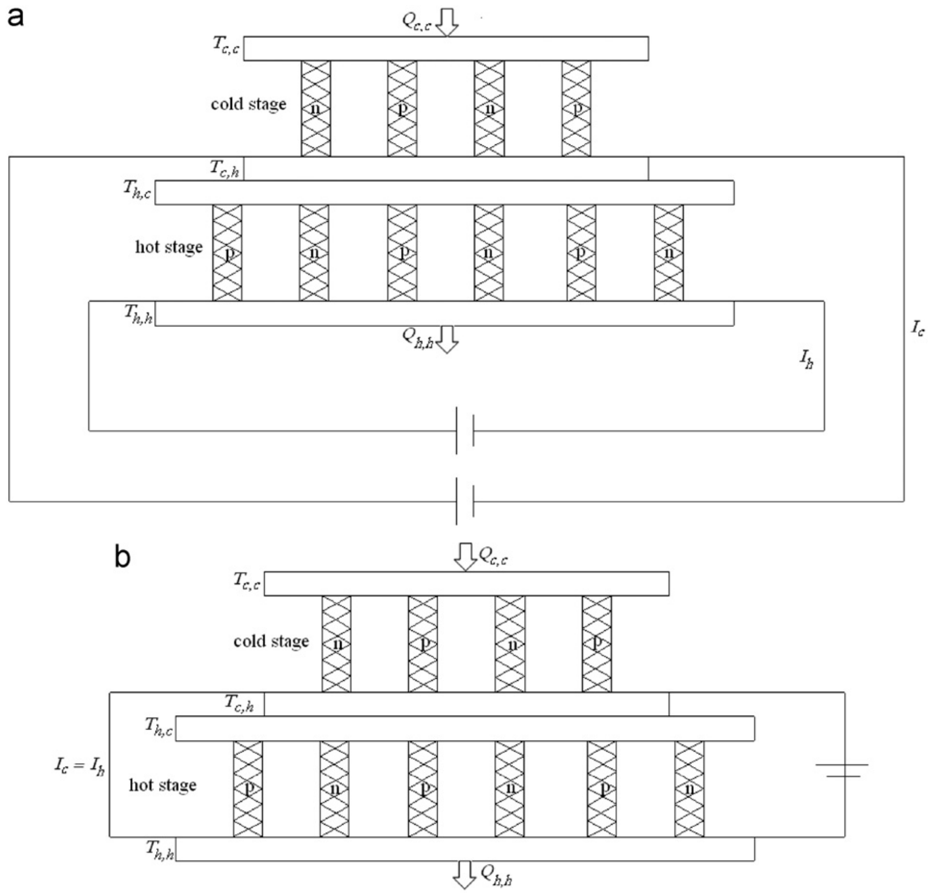

Fig. 1. Two stage TEC. (a) Electrically separated and (b) electrically connected in series.

2. Previous work on TECs optimization

Several investigators had used different methodologies considering different objective functions to optimize the TECs design. Chen et al. (2002) carried out the optimal performance comparison of single and two-stage TE refrigeration systems. The authors had calculated the maximum COP and rate of refrigeration and optimized the internal structure parameter of the TE device. Xuan et al. (2002a) carried out the optimization of a two stage TEC with two design configurations. The authors had found out the optimum ratio of the number of TE modules between the stages and optimum ratio of the electric current between stages for maximization of cooling capacity and COP of TEC. Xuan (2002) and Xuan et al. (2002b) carried out the performance analysis of a two stage TEC with three design configurations. The authors had considered the maximum cooling capacity, maximum COP and the maximum temperature difference of the two-stage TEC. Chen et al. (2006) carried out the parametric optimum design of a TE device. The authors had calculated the maximum COP and rate of refrigeration of the system and determined the optimal operating region of the system. Pan et al. (2007) carried out the performance analysis and parametric optimization of a multi-couple TE refrigerator. The authors had determined the optimal operating-state of the COP for a TE refrigeration device.

Cheng and Lin (2005) used a genetic algorithm for geometric optimization of TEC. The authors had considered maximization of cooling capacity as an objective function and determined the optimum value of structure parameter of TE modules. Cheng and Shih (2006) used GA for maximizing the cooling capacity and COP of a two-stage TEC. The authors had considered the effect of thermal resistance and determined the optimum value of input current and number of TE modules for two different design configurations of TEC. Abramzon (2007) used multi-start adaptive random search method for the numerical optimization of the TEC. The author had considered maximization of total cooling rate of the TEC as an objective function. Yu et al. (2007) analyzed the optimum configuration of two stage TE modules. The authors had investigated the influence of different parameters on the cooling performance of the TE modules. Chen et al. (2009) analyzed the performance of a two-stage TE heat pump system driven by a two-stage TE generator. The authors had optimized the allocations of the TE element pairs among the two TE generators and the two TE heat pumps for maximizing the heating load and COP respectively. Several other researchers (Lai et al., 2004; Chen et al., 2005; Chen et al., 2008; Meng et al., 2009) investigated the two stage TECs for optimization of COP or for optimum allocation of TE module.

So far, only GA is used for the optimization of TECs. Moreover, only single objective optimization of TECs was carried out by previous researchers. Considering this fact, the main objectives of this work are: (i) multi-objective optimization of the influential parameters of a two stage TEC using the basic TLBO and modified TLBO algorithms and (ii) to demonstrate the effectiveness of the modified TLBO algorithm for multi-objective optimization of the TEC. So far, TLBO algorithm has not been tried or experimented for the multi-objective optimization of TECs. In this paper, ability of the algorithm is demonstrated using an example. The optimization results obtained by using TLBO and the modified TLBO algorithms are compared with those obtained by using GA for the same example considered by previous researchers.

3. Teaching–learning-based optimization (TLBO)

TLBO is a teaching–learning process inspired algorithm proposed recently by Rao et al. (2011) based on the effect of influence of a teacher on the output of learners in a class. The algorithm mimics the teaching–learning ability of teacher and learners in a class room. Teacher and learners are the two vital components of the algorithm. In this optimization algorithm, a group of learners is considered as population and different design variables are considered as different subjects offered to the learners and the learners' result is analogous to the 'fitness' value of the optimization problem. In the entire population, the best solution is considered as the teacher. The working of TLBO is divided into two parts, 'teacher phase' and 'learner phase'. Working of both the phases is explained below.

3.1. Teacher phase

It is the first part of the algorithm where learners learn through the teacher. During this phase a teacher tries to increase the mean result of the class room from any value M_1 to his or her level (i.e. T_A). But practically it is not possible and a teacher can move the mean of the class room M_1 to any other value M_2 which is better than M_1 depending on his or her capability. Consider M_j be the mean and T_i be the teacher at any iteration i. Now T_i will try to improve existing mean M_j towards him or her so the new mean will be designated as M_{new} and the difference between the existing mean and new mean is given by

$$\text{Difference\_Mean}_i = r_i(M_{new} - T_F M_j) \tag{1}$$

where T_F is the teaching factor which decides the value of mean to be changed, and r_i is the random number in the range [0, 1]. Value of T_F can be either 1 or 2 which is a heuristic step and it is decided randomly with equal probability as

$$T_F = \text{round}[1 + \text{rand}(0,1)\{2-1\}] \tag{2}$$

Based on this Difference_Mean, the existing solution is updated according to the following expression:

$$X_{new,i} = X_{old,i} + \text{Difference\_Mean}_i \tag{3}$$

3.2. Learner phase

It is the second part of the algorithm where learners increase their knowledge by interaction among themselves. A learner interacts randomly with other learners for enhancing his or her knowledge. A learner learns new things if the other learner has more knowledge than him or her. Mathematically the learning phenomenon of this phase is expressed below.

At any iteration i, considering two different learners X_i and X_j where $i \neq j$

$$X_{new,i} = X_{old,i} + r_i(X_i - X_j) \text{ If } f(X_i) < f(X_j) \tag{4}$$

$$X_{new,i} = X_{old,i} + r_i(X_j - X_i) \text{ If } f(X_j) < f(X_i) \tag{5}$$

Accept X_{new} if it gives better function value.

4. Modified TLBO algorithm

In the basic TLBO algorithm, result of the learners is improved either by a single teacher or by interacting with other learners. However, the aspect of self-motivated learning of the learners is not considered in the basic TLBO algorithm. Moreover, the teaching factor in the basic TLBO algorithm is either 2 or 1 which reflects two extreme circumstances where learner learns either everything or nothing from the teacher. In this system, teacher has to give more effort to improve the result of learners. During the course of optimization, this situation results in slower convergence rate of optimization problem. So considering this fact, to speed up the search process and to improve the convergence rate, some modifications have been introduced in the basic TLBO algorithm.

4.1. Number of teachers

One of the modifications in the TLBO algorithm is introducing more than one teacher for learners. In the basic version of TLBO there is only one teacher who teaches the learners and tries to improve the mean result of the class. In this system of learning if the class contains more number of below-average students, then the teacher has to put more effort to improve their results and even with this effort there might not be any apparent improvement in the results. In the optimization algorithm this fact results in more number of function evaluations to reach at the optimum solution and yields poor convergence rate. In order to overcome this issue, the basic TLBO algorithm is modified by introducing more than one teacher to the learners. By means of this modification, entire class is split into different groups of learners as per their level (i.e. results) and individual teacher is assigned to individual group of learners. Now, each teacher tries to improve the results of his or her assigned group and if the level (i.e. results) of the group reaches up to the level of the assigned teacher then this group is assigned to a better teacher. Thus the learners learn as per their ability from different teachers instead from one teacher which increases the possibility of improvement in the results of learners. During the course of optimization this situation increases the exploration and exploitation capacity of the algorithm. Mathematically, this modification is explained in the implementation steps of the algorithm.

4.2. Adaptive teaching factor

Another modification is related to the teaching factor (T_F) of the basic TLBO algorithm. Teaching factor decides the value of mean result of the learners to be changed (i.e. decide the update value of the objective function during the optimization). In the basic TLBO, the decision of the teaching factor is a heuristic step and it can be either 1 or 2. This practice is corresponding to situation where learners learn nothing from the teacher or learn all the things from the teacher respectively. Thus during the course of optimization the objective function updates only with these two possibilities. In actual teaching–learning phenomenon the learners may learn in any proportion from the teacher and it means that the teaching factor is not always at its end state for learners but varies in-between also. Considering this fact, teaching factor is modified in the present work. This modification increases the exploration and exploitation capacity of the basic TLBO algorithm. In the optimization algorithm the lower value of T_F allows the fine search in small steps but causes slow convergence. A larger value of T_F speeds up the search but it reduces the exploration capability. The modified teaching factor is defined as

$$T_{F_i} = \frac{M_{D,i}}{M\_\text{new}_{D,i}} \quad D = 1,2,\ldots,D_n \;\; i = 1,2,\ldots N_G \tag{6}$$

where, D_n is the number of design variables and NG is the number of generations. $M_{D,i}$ is the mean of the learners in any subject at iteration i and $M\_\text{new}_{D,i}$ is the position of the teacher for the same subject at iteration i. Thus in the modified TLBO algorithm the teaching factor varies automatically during the search. Automatic tuning of T_F improves the performance of the algorithm.

4.3. Self-motivated learning

In the basic TLBO algorithm, the results of the students are improved either by learning from the teacher or by interacting

with the other students. However, it is also possible that students are self-motivated and improve their knowledge by self-learning. Thus the self-learning aspect to improvise the knowledge is considered in the modified TLBO algorithm which again increases the exploration and exploitation capacity of the basic TLBO algorithm. Mathematical formulation of this modification is given in the implementation steps of the algorithm.

Implementation steps of the modified TLBO algorithm is summarized below

Step 1: define the optimization problem as, Minimize or Maximize $f(X)$

Subject to $X_i x_i = 1,2,\ldots,D_n$

where $f(X)$ is the objective function, and X is a vector for design variables (D_n) within its upper and lower limits.

Step 2: initialize the population (i.e. learners) (P_n) and design variables of the optimization problem (i.e. number of subjects offered to the learners) (D_n) with random generation and evaluate them.

Step 3: select the best learner (i.e. best solution, $f(X)_{best}$) as a chief teacher and assign him or her the first rank (i.e. $f(X)_1$). He or she will act as a chief teacher for that iteration.

$$X_{1,teacher} = f(X)_1$$

Step 4: based on the chief teacher, the other teachers (T) are selected as

$$f(X)_s = f(X)_1 - \text{rand} * f(X)_1 \quad s = 2,3,\ldots T$$

(If the equality is not met, select the $f(X)s$ closer to the value calculated above)

Rank the teachers in the ascending order of $f(X)s$ value.

$$X_{s,teacher} = f(X)_s \quad s = 2,3,\ldots T$$

Step 5: assign the learners to the teachers according to their fitness value as

For $L = 1$: P_n.
If $f(X)_1 \geq f(X)_L > f(X)_2$
Assign the learner $f(X)_L$ to teacher 1 (i.e. $f(X)_1$).
Else If $f(X)_2 \geq f(X)_L > f(X)_3$
Assign the learner $f(X)_L$ to teacher 2 (i.e. $f(X)_2$).
⋮
Else If $f(X)_{T-1} \geq f(X)_L > f(X)_T$
Assign the learner $f(X)_L$ to teacher '$T-1$' (i.e. $f(X)_{T-1}$).
Else
Assign the learner $f(X)_L$ to teacher 'T'
End

(Above procedure is for maximization problem; the procedure is reversed for minimization problem, i.e. if $f(X)_1 \geq f(X)_L > f(X)_2$, assign the learner L to teacher 2)

Step 6: calculate mean of each group of learners in each subject (i.e. $M_{s,D}$). Also in each group, the teacher of that group acts as a new mean for the iteration and tries to shift the mean from $M_{s,D}$ towards $X_{s,teacher}$.

$$M\_new_{s,D} = X_{s,D} \quad s = 1,2,\ldots T, \quad D = 1,2,\ldots D_n$$

Step 7: evaluate the difference between the current mean and the best mean for each group by utilizing the teaching factor (T_F) as

$$\text{Difference\_Mean}_{s,D} = r(M\_new_{s,D} - T_{F_s} M_{s,D}) \quad s = 1,2\ldots,T,$$

$$D = 1,2\ldots,D_n \quad T_{F_s} = \frac{M_{s,D}}{M\_new_{s,D}}$$

Step 8: for each group, update the learners' knowledge with the help of teacher's knowledge according to

$$X_{new,D} = X_{old,D} + \text{Difference\_Mean}_{s,D}$$

Step 9: for each group, update the learners' knowledge by utilizing the knowledge of some other learner of the same group as per Eqs. (4) and (5).

Step 10: learners of each group tries to improvise their knowledge by self-learning according to

$$X_{new,D} = X_{old,D} + r(X_{max,D} - X_{min,D})$$

where, $X_{max,D}$ and $X_{min,D}$ are the maximum and minimum value of the respective design variables.

Step 11: combine all the groups.

Step 12: repeat the procedure from step 3 to 11 till the termination criterion is met.

This algorithm has been applied for the multi-objective optimization of a two stage TEC. The next section describes the thermal modeling of two stage TECs.

5. Thermal modeling of two stage TECs

Based on the work of Cheng and Shih (2006), thermal model of the two stage TECs is formulated as described below.

The cascade two stage TECs are stacked one on the top of the other (as shown in Fig. 1). Here in this arrangement the top stage is the cold stage and the bottom stage is the hot stage. In Fig. 1, $Q_{c,c}$ and $Q_{h,h}$ are the cooling capacities of the cold side of the cold stage and the heat rejected at the hot side of hot stage respectively. $T_{c,c}$, $T_{c,h}$, $T_{h,c}$ and $T_{h,h}$ are the temperatures of the cold side of the cold stage, hot side of the cold stage, cold side of the hot stage and hot side of the hot stage respectively. I_c and I_h are the input currents to the cold stage and the hot stage respectively. n And p stand for n-type and p-type TE modules respectively. The COP of the two stage TECs is given by

$$\text{COP} = \frac{Q_{c,c}}{Q_{h,h} - Q_{c,c}} \tag{7}$$

where, $Q_{c,c}$ and $Q_{h,h}$ are obtained by heat balance at relevant junction of TECs

$$Q_{c,c} = \frac{N_t}{r+1}\left[\alpha_c I_c T_{c,c} - \frac{1}{2} I_c^2 R_c - K_c (T_{c,h} - T_{c,c})\right] \tag{8}$$

$$Q_{h,h} = \frac{N_t r}{r+1}\left[\alpha_h I_h T_{h,h} + \frac{1}{2} I_h^2 R_h - K_h (T_{h,h} - T_{h,c})\right] \tag{9}$$

where, N_t is the total number of TE modules of two stages and r is the ratio of the number of TE modules between the hot stage (N_h) to the cold stage (N_c). α, R and K are the Seebeck coefficient, electrical resistance and thermal conductance of the cold stage and the hot stage respectively and their relation to TE material properties is given by

$$\alpha_i = (\alpha_{i,p} - \alpha_{i,n})_{T_{i,ave}} \tag{10}$$

$$R_i = \frac{[\rho_{i,p} + \rho_{i,n}]_{T_{i,ave}}}{G} \tag{11}$$

$$K_i = [k_{i,p} + k_{i,n}]_{T_{i,ave}} G \tag{12}$$

where, subscript i stands for the cold side (c) and the hot side (h) of TEC; subscript *ave* indicates the average value and subscripts p and n indicate the properties of p and n-type TE modules. G is the structure parameter of the TE modules and indicates the ratio of cross section area to the length of TE modules. ρ And k are the electric resistivity and thermal conductivity of the TE material

respectively. As the material properties are considered to be dependent on the average temperature of the cold side and hot side of each stage, their values are calculated by the following correlation (Cheng and Shih, 2006):

$$\alpha_{i,p} = -\alpha_{i,n} = (22{,}224 + 9300.6T_{i,ave} - 0.9905T_{i,ave}^2)10^{-9} \tag{13}$$

$$\rho_{i,p} = \rho_{i,n} = (5112 + 163.4T_{i,ave} + 0.6279T_{i,ave}^2)10^{-10} \tag{14}$$

$$k_{i,p} = k_{i,n} = (62{,}605 - 277.7T_{i,ave} + 0.4131T_{i,ave}^2)10^{-4} \tag{15}$$

The total thermal resistance (RS_t) existing between the interface of the TECs is given by

$$RS_t = RS_{sprd} + RS_{cont} \tag{16}$$

where, RS_{sprd} and RS_{cont} are the spreading resistance and contact resistance between the interface of the two TECs respectively.

Based on the work of Lee et al. (1995) and Cheng and Shih (2006), the spreading resistances between the interface of the two TECs are calculated from the following equation:

$$RS_{sprd} = \frac{\psi_{max}}{k_{h,s}\mathrm{rad}_{c,s}\sqrt{\pi}} \tag{17}$$

where, $\mathrm{rad}_{c,s}$ is the equilibrium radius of the substrates of the cold stage and $k_{h,s}$ is the thermal conductivity of the substrate of the hot stage. The detailed explanation related to the equilibrium radius is available in the work of Lee et al. (1995). However, it is calculated by the following equation:

$$\mathrm{rad}_{c,s} = \sqrt{\frac{(2aN_t/r+1)}{\pi}} \tag{18}$$

where, factor $2a$ represents the linear relationship between the cross-sectional area of the substrate and the TE modules (Cheng and Shih, 2006).

The dimensionless parameter ψ_{max} of Eq. (17) is given by

$$\psi_{max} = \frac{\varepsilon\tau}{\sqrt{\pi}} + \frac{1}{\sqrt{\pi}}(1-\varepsilon)\varphi \tag{19}$$

where, ε and τ are the dimensionless parameters and are calculated by

$$\varepsilon = \frac{\mathrm{rad}_{c,s}}{\mathrm{rad}_{h,s}} = \sqrt{\frac{1}{r}} \tag{20}$$

$$\tau = \frac{s_{h,s}}{\mathrm{rad}_{h,s}} \tag{21}$$

where, $\mathrm{rad}_{h,s}$ is the equilibrium radius of the substrate of the hot stage and $S_{h,s}$ is the substrate thickness of the hot stage respectively and given by

$$\mathrm{rad}_{h,s} = \sqrt{\frac{(2aN_t r/r+1)}{\pi}} \tag{22}$$

The dimensionless parameter φ of Eq. (19) is given by

$$\varphi = \frac{\tanh(\lambda \times \tau) + \lambda/\mathrm{Bi}}{1 + \lambda/\mathrm{Bi}\tanh(\lambda \times \tau)} \tag{23}$$

where, Bi is the *Biot* number and its value is infinity i.e. ($\mathrm{Bi} = \infty$) for isothermal cold side of the hot stage.

The dimensionless parameter λ of Eq. (23) is given by (Cheng and Shih, 2006)

$$\lambda = \pi + \frac{1}{\varepsilon\sqrt{\pi}} \tag{24}$$

The contact thermal resistance (RS_{cont}) at the interface of the two TECs is calculated by

$$RS_{cont} = \frac{RS_j}{(2aN_t/r+1)} \tag{25}$$

where, Rs_j is the joint resistance at the interface of two TECs.

The heat rejected at the hot side of the cold stage ($Q_{c,h}$) and cooling capacity at the cold side of the hot stage ($Q_{h,c}$) are obtained by considering the heat balance at the interface of TECs

$$Q_{c,h} = \frac{N_t}{r+1}\left[\alpha_c I_c T_{c,h} + \frac{1}{2}I_c^2 R_c - K_c(T_{c,h} - T_{c,c})\right] \tag{26}$$

$$Q_{h,c} = \frac{N_t r}{r+1}\left[\alpha_h I_h T_{h,c} - \frac{1}{2}I_h^2 R_h - K_h(T_{h,h} - T_{h,c})\right] \tag{27}$$

As the hot side of the cold stage and cold side of the hot stage are at the interface so $Q_{c,h} = Q_{h,c}$, but due to the thermal resistance at the interface, the temperature of the both side are not same. The relation between both these temperatures is given by (Cheng and Shih, 2006)

$$T_{h,c} = T_{c,h} + RS_t Q_{c,h} \tag{28}$$

The next section describes the objective function formulation based on this thermal model of two stage TECs.

6. Multi-objective optimization and formulation of objective functions

Multi-objective optimization has been defined as finding a vector of decision variables while optimizing (i.e. minimizing or maximizing) several objectives simultaneously, with a given set of constraints. In the present work, two such objectives namely maximizing the cooling capacity and maximizing the COP of the two stage TECs are considered simultaneously for multi-objective optimization.

The first objective is to maximize the cooling capacity of a two stage TEC as given by Eq. (29)

$$Z_1 = \mathrm{Maximize}\,Q_{c,c}(X), \quad X = [x_1, x_2, \ldots x_{D_n}], \quad x_{i,min} \le x_i \le x_{i,max}, \quad i = 1,2,\ldots D_n \tag{29}$$

Subject to the set of constraints (m)

$$g_j(X) \le 0, \quad j = 1,2\ldots m \tag{30}$$

The second objective is to maximize the COP of a two stage TEC as given by Eq. (31)

$$Z_2 = \mathrm{Maximize}\,\mathrm{COP}(Y), \quad Y = [y_1, y_2, \ldots y_{D_n}], \quad y_{i,min} \le y_i \le y_{i,max}, \quad i = 1,2,\ldots D_n \tag{31}$$

Subject to a set of constraints (m)

$$g_j(Y) \le 0, \quad j = 1,2\ldots m \tag{32}$$

The above mentioned single objective functions are put together for multi-objective optimization. The normalized multi-objective function (Z) is formulated considering different weight factors to both the objectives and is given by the following equation:

$$\mathrm{Maximize}\,Z = w_1(Z_1/Z_{1,max}) + (1-w_1)(Z_2/Z_{2,max}) + \sum_{j=1}^{m} R1(g_j(X))^2 + \sum_{j=1}^{m} R1(g_j(Y))^2 \tag{33}$$

where, w_1 is weight factor for the first objective function. $Z_{1,max}$ and $Z_{2,max}$ are the maximum values of the objective functions Z_1 and Z_2 respectively when these objectives are considered independently. The last two terms in Eq. (33) takes into account the constraints violation. $R1$ is the penalty parameter having a large value (1000 in the present work). The value of weight factor w_1 can be decided by the designer and it is between 0 and 1. Any value of w_1 between 0 and 1 gives the relative importance to the individual objective function and the algorithm finds the design variables as per this relative importance. Based on the obtained design variables, Z_1 and Z_2 are obtained which represent one solution of the Pareto front. Similarly, by considering the different

value of w_1 a set of optimum solutions, called Pareto solutions is obtained, each of which is a trade-off between the considered objective functions. In the present work we considered the value of w_1 from 0 to 1 in steps of 0.05.

Now an example is considered to demonstrate the effectiveness of the modified TLBO algorithm for the optimization of two stage TECs.

7. Application example of two stage TEC

The effectiveness of the modified TLBO algorithm is assessed by analyzing an example of two stage TECs which was earlier analyzed by Cheng and Shih (2006) using GA. A Two stage TEC used to produce temperature of 210 K at the cold stage when its hot stage is maintained at a temperature of 300 K is needed to be optimized for maximum cooling capacity and maximum COP. The total number of TE modules of the two stages is 50 and the ratio of cross sectional area to the length of TE modules is 0.0018 m. Thermal resistance is present at the interface of TEC. Alumina having thermal conductivity 30 W/m K is acting as a substrate to take into account the spreading resistance. The thickness of the substrate is 1 mm. To take into account the contact resistance between the two stages, the joint resistance is varied between 0.02 to 2 cm^2 K/W. The property values of TE material are considered to be temperature dependent. Moreover, the two stage TECs, electrically separated and electrically connected in series as shown in Fig. 1 are considered for the optimization.

Following inequality constraints which are bound by lower and upper limits of the design variables are considered in the present work of TECs optimization

$$4 \le I_h \le 11 \quad (34)$$

$$4 \le I_c \le 11 \quad (35)$$

$$2 \le r \le 7.33 \quad (36)$$

Number of trial runs of the TLBO and modified TLBO algorithm are performed with different population sizes and number of generations for setting the best strategy. Finally, a population size of 15 with number of generations 30 is set for both the algorithms. With this strategy one hundred independent runs of the algorithms are performed for each objective function and average results of one hundred independent runs is reported in the present work.

7.1. Single objective consideration

To understand the working of modified TLBO algorithm, stepwise procedure for the implementation of modified TLBO algorithm for one generation is given in the appendix. For demonstration of the procedure, maximization of cooling capacity of electrically separated TEC is considered as an objective function.

Table 1 shows the optimized parameters of the considered example obtained by using the TLBO and the modified TLBO approach for maximum cooling capacity as well as maximum COP when the considered two stage TEC is electrically separated and its comparison with the optimized parameters obtained by Cheng and Shih (2006) using the GA approach. When the joint resistance is 0.02 cm^2 K/W, present approach using the TLBO and the modified TLBO results in such combination of input current and TE module which increases the cooling capacity by 3.84% as compared to the GA approach suggested by Cheng and Shih (2006). Also as the joint resistance increases from 0.02 to 0.2 and then 2 cm^2 K/W, the increment in cooling capacity is 5.32% and 7.18% respectively as compared to the GA approach. Similarly, for the maximum COP consideration, the present approaches yields 1.1%, 4.29% and 7.21% higher COP as compared to the GA approach when the joint resistance is 0.02, 0.2 and 2 cm^2 K/W respectively.

Table 2 shows the comparison of the optimized parameters for a two stage TEC electrically connected in series. In this case also, the increment in cooling capacity is corresponding to 2.45%, 8.1% and 7.2% when the joint resistance is 0.02, 0.2 and 2 cm^2 K/W respectively as compared to the GA approach considered by Cheng and Shih (2006). Similarly, for the maximum COP consideration the present approaches result in 0.5%, 7.5% and 5.41%

Table 1
Comparison of the two-stage TEC (electrically separated).

	GA (Cheng and Shih, 2006)		PSO		ABC		TLBO		Modified TLBO	
	Max $Q_{c,c}$	Max COP	Max $Q_{c,c}$	Max COP	Max $Q_{c,c}$	Max COP	Max $Q_{c,c}$	Max COP	Max $Q_{c,c}$	Max COP
RS_j = 0.02 cm^2 K/W										
I_h (A)	8.613	6.611	9.2671	7.0044	9.3978	6.7299	9.3077	6.7299	9.3077	6.7299
I_c (A)	7.529	7.592	7.8411	7.3077	7.6967	7.581	7.7146	7.581	7.7146	7.581
r	5.25	6.143	5.25	5.25	5.25	6.143	5.25	6.143	5.25	6.143
N_c	8	7	8	8	8	7	8	7	8	7
$Q_{c,c}$ (W)	0.755	–	0.7833	0.6141	0.7837	0.5968	0.784	0.5968	0.784	0.5968
COP	–	0.019	0.015	0.0191	0.015	0.0192	0.015	0.0192	0.015	0.0192
Time (s)	–	–	65.11	65.96	73.93	74.64	86.44	87.23	91.37	92.58
RS_j = 0.2 cm^2 K/W										
I_h (A)	8.652	6.769	9.3278	6.5338	9.3278	6.5338	9.3278	6.5338	9.3278	6.5338
I_c (A)	7.805	7.465	8.0121	7.8165	8.0121	7.8165	8.0121	7.8165	8.0121	7.8165
r	5.25	6.143	5.25	6.143	5.25	6.143	5.25	6.143	5.25	6.143
N_c	8	7	8	7	8	7	8	7	8	7
$Q_{c,c}$ (W)	0.838	–	0.8826	0.6544	0.8826	0.6544	0.8826	0.6544	0.8826	0.6544
COP	–	0.021	0.0168	0.0219	0.0168	0.0219	0.0168	0.0219	0.0168	0.0219
Time (s)	–	–	65.89	66.62	74.58	75.41	87.19	88.08	92.28	93.47
RS_j = 2 cm^2 K/W										
I_h (A)	9.29	5.204	9.413	4.8169	9.609	4.5779	9.609	4.4163	9.609	4.4163
I_c (A)	9.41	9.889	10.8829	10.2275	11	10.4732	11	10.722	11	10.722
r	4.556	5.25	4.556	6.143	4.556	7.333	4.556	7.333	4.556	7.333
N_c	9	8	9	7	9	6	9	6	9	6
$Q_{c,c}$ (W)	2.103	–	2.25	1.3329	2.254	1.2381	2.254	1.201	2.254	1.201
COP	–	0.061	0.0406	0.0647	0.0393	0.0652	0.0393	0.0654	0.0393	0.0654
Time (s)	–	–	67.19	68.14	75.94	76.87	88.26	89.19	93.64	94.83

Table 2
Comparison of the two-stage TEC (electrically connected in series).

	GA (Cheng and Shih, 2006)		PSO		ABC		TLBO		Modified TLBO	
	Max $Q_{c,c}$	Max COP	Max $Q_{c,c}$	Max COP	Max $Q_{c,c}$	Max COP	Max $Q_{c,c}$	Max COP	Max $Q_{c,c}$	Max COP
RS_j = 0.02 cm² K/W										
I_h (A)	8.415	7.27	8.5737	7.1558	8.5737	7.1558	8.5737	7.1558	8.5737	7.1558
I_c (A)	8.415	7.27	8.5737	7.1558	8.5737	7.1558	8.5737	7.1558	8.5737	7.1558
r	6.143	5.25	6.143	5.25	6.143	5.25	6.143	5.25	6.143	5.25
N_c	7	8	7	8	7	8	7	8	7	8
$Q_{c,c}$ (W)	0.73	–	0.7479	0.6405	0.7479	0.6405	0.7479	0.6405	0.7479	0.6405
COP	–	0.019	0.0159	0.0191	0.0159	0.0191	0.0159	0.0191	0.0159	0.0191
Time (s)	–	–	47.18	47.61	54.67	55.29	63.43	64.14	66.17	66.98
RS_j = 0.2 cm² K/W										
I_h (A)	8.663	7.135	8.5978	7.4962	8.7375	7.4962	8.7375	7.1681	8.7375	7.1681
I_c (A)	8.663	7.135	8.5978	7.4962	8.7375	7.4962	8.7375	7.1681	8.7375	7.1681
r	6.143	5.25	6.143	5.25	6.143	5.25	6.143	6.143	6.143	6.143
N_c	7	8	7	8	7	8	7	8	7	8
$Q_{c,c}$ (W)	0.818	–	0.8328	0.7157	0.8338	0.7157	0.8338	0.7098	0.8338	0.7098
COP	–	0.02	0.0177	0.0213	0.0172	0.0213	0.0172	0.0215	0.0172	0.0215
Time (s)	–	–	47.59	48.02	55.18	56.07	64.02	64.97	66.91	67.89
RS_j = 2 cm² K/W										
I_h (A)	9.482	7.133	9.7236	7.305	10.1207	7.305	10.387	7.305	10.387	7.305
I_c (A)	9.482	7.133	10.4581	7.305	10.1207	7.305	10.387	7.305	10.387	7.305
r	4	4.555	4	3.546	4	3.546	4.556	3.546	4.556	3.546
N_c	10	9	10	11	10	11	9	11	9	11
$Q_{c,c}$ (W)	2.123	–	2.2614	1.6947	2.273	1.6947	2.276	1.6947	2.276	1.6947
COP	–	0.048	0.0398	0.0506	0.0374	0.0506	0.0354	0.0506	0.0354	0.0506
Time (s)	–	–	48.84	49.56	56.98	57.81	65.49	66.15	68.51	69.34

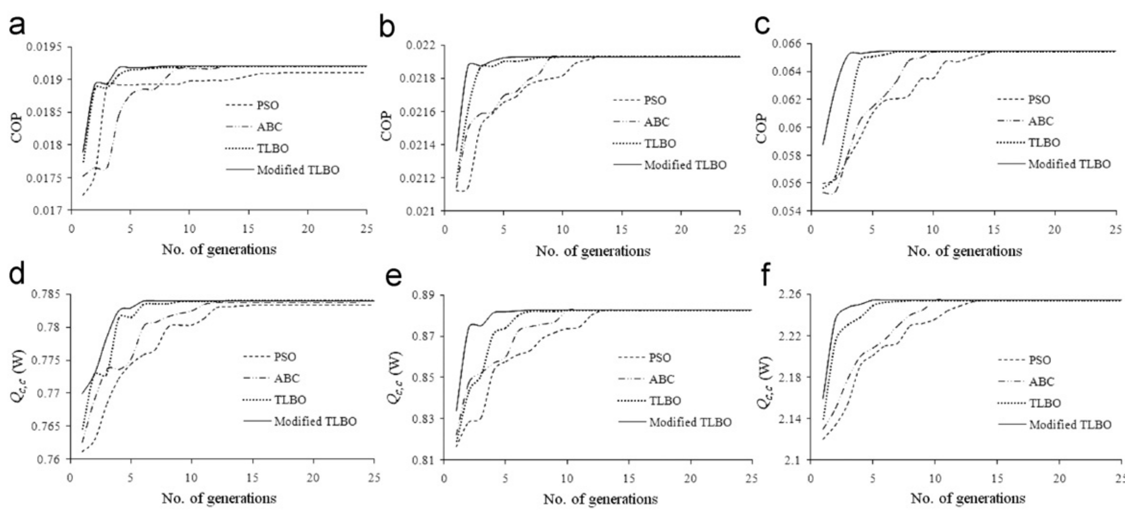

Fig. 2. Convergence of the PSO, ABC, TLBO and modified TLBO algorithms for single objective optimization of electrically separated TEC. (a and d) RS_j=0.02 cm² K/W, (b and e) RS_j=0.2 cm² K/W and (c and f) RS_j=2 cm² K/W.

higher COP as compared to the GA approach when the joint resistance is 0.02, 0.2 and 2 cm² K/W respectively. Moreover, in order to identify the effectiveness of the presented algorithm, Particle Swarm optimization (PSO) and Artificial Bee Colony (ABC) algorithms are also applied to the considered problem and comparative results are shown in Tables 1 and 2 for TEC electrically separated as well as connected in series respectively.

Figs. 2 and 3 show the convergence of the TLBO, modified TLBO, ABC and PSO algorithms for single objective optimization when the considered TEC is electrically separated and electrically in series respectively. For the fair comparison among all the algorithms, PSO and ABC are also applied with the population size of 15 and number of generations 30. It is observed from Figs. 2 and 3 that the modified algorithm performs better in terms of convergence.

7.2. Multi-objective consideration

The results of single objective optimization for maximum cooling capacity and maximum COP reveal that higher cooling capacity accompanies the lower COP and vice versa which reflects the necessity of multi-objective optimization for two stage TECs. Eq. (33) represents the normalized objective function for multi-objective optimization. Fig. 4 shows the Pareto optimal curve obtained by using the modified TLBO algorithm for multi-objective optimization when the considered two stage TECs are electrically separated. As seen from Fig. 4(a)–(c) that for different values of joint resistance the maximum cooling capacity observed at design point E where the COP is minimum. On the other hand, the maximum COP occurs at design point A where the cooling

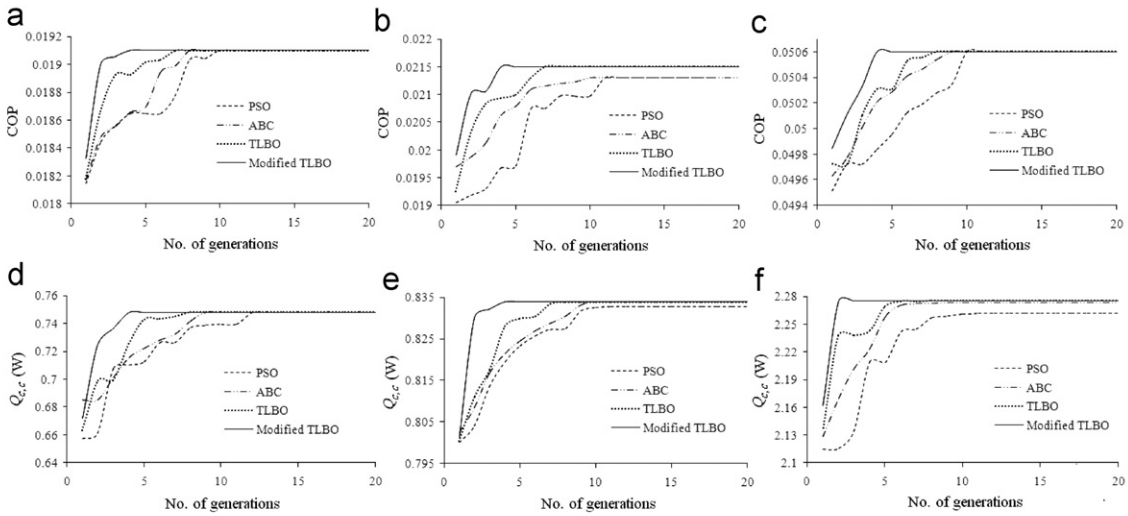

Fig. 3. Convergence of the PSO, ABC, TLBO and modified TLBO algorithms for single objective optimization of electrically series connected TEC. (a and d) $RS_j=0.02$ cm^2 K/W, (b and e) $RS_j=0.2$ cm^2 K/W and (c and f) $RS_j=2$ cm^2 K/W.

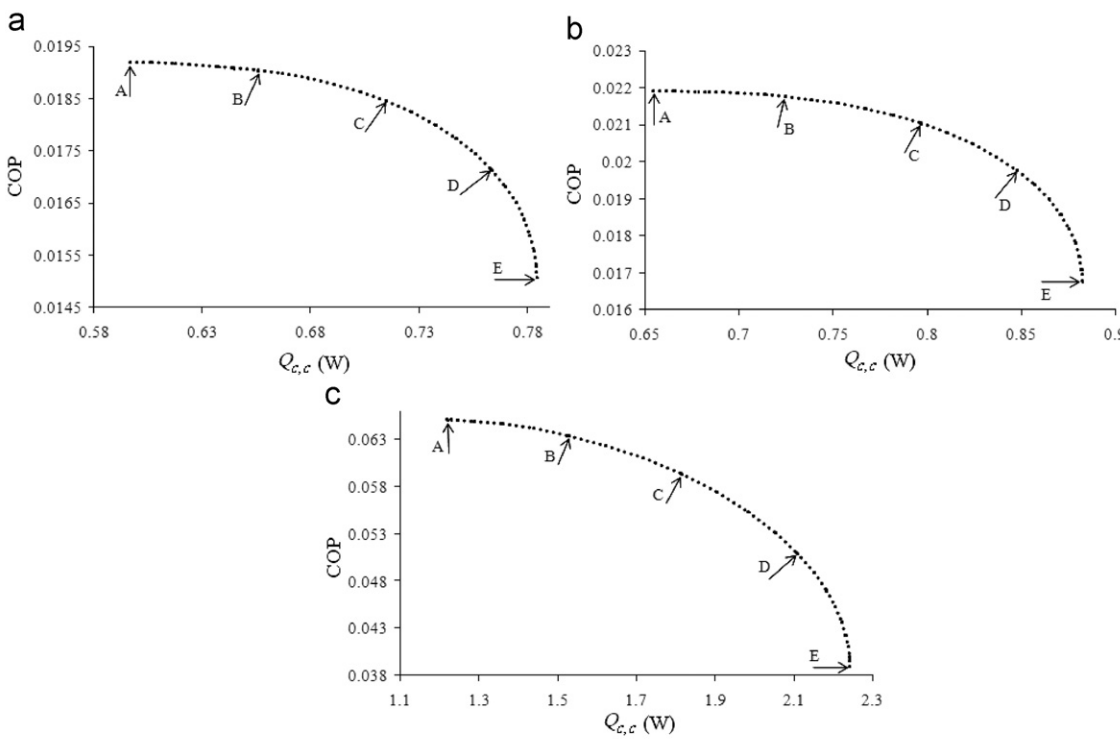

Fig. 4. The distribution of Pareto-optimal points solutions for electrically separated TEC using the modified TLBO algorithm. (a) $RS_j=0.02$ cm^2 K/W, (b) $RS_j=0.2$ cm^2 K/W and (c) $RS_j=2$ cm^2 K/W.

capacity has minimum value. Specifications of five sample design points A–E in Pareto optimal fronts for different values of joint resistance are listed in Table 3. It is observed from Fig. 4(a)–(c) and Table 3 that by properly modulating the input current of hot stage and cold stage as well as TE module of each stage, the cooling capacity and COP of the two stage TEC increases with the increase in joint resistance.

Fig. 5 represents the Pareto optimal curve obtained by using the modified TLBO algorithm for the two stage TECs electrically connected in series. Looking at the Pareto front obtained for different values of joint resistance it is found that the maximum cooling capacity exists at design point E where the COP is lowest. On the other hand, the maximum COP occurs at design point A where the cooling capacity has minimum value. Table 4 shows the specifications of sample design points A–E in Pareto optimal fronts for different values of the joint resistance.

The effect of number of teachers on the fitness value and convergence rate of the multi-objective function is shown graphically in Figs. 6 and 7 when the TEC is electrically separated as well as in series respectively. Design point D of the Pareto optimal curves is considered for demonstrating the effect of teachers. It is observed from Figs. 6 and 7 that as the number of teachers increases, the algorithm performs better in terms of the convergence rate. However, after sufficient number of teachers, any increment in the number of teachers does not improve the performance of the algorithm.

In order to identify the effect of input current and TE modules of each stage on the cooling capacity and COP, design point D of the Pareto optimal curves is considered. Fig. 8 shows the effect of I_h on the cooling capacity and COP when the considered two stage TEC is electrically separated. It is observed from Fig. 8(a)–(c) that with the increase in I_h the cooling capacity and COP of TEC increases and reaches a maximum value. With further increase in I_h, the cooling capacity and COP decrease. Also it is observed that as the joint resistance is increased from 0.02 to 2 cm^2 K/W, the value of I_h at which the cooling capacity and COP become maximum is reduced. This behavior reflects that the sensitivity of I_h for cooling capacity and COP increases with the increase in joint resistance.

Fig. 9 shows the effect of I_c on the cooling capacity and COP when the considered two stage TEC is electrically separated. It is observed from Figs. 9 and 10 that when joint resistance is 0.02 cm^2 K/W the sensitivity of I_c for cooling capacity and COP is more comparable to I_h. But as the joint resistance is increased the sensitivity of I_h is increased while sensitivity of I_c is decreased. Fig. 10 shows the effect of ratio of TE modules between the hot stage and the cold stage (r) when the considered two stage TEC is

Table 3
Optimal output variables for A to E Pareto optimal front shown in Fig. 5.

Output variable	Design point				
	A	B	C	D	E
RS_j=0.02 cm^2 K/W					
I_h (A)	6.7299	7.4285	8.0476	8.7347	9.3077
I_c (A)	7.581	7.4018	7.5229	7.6351	7.7146
r	6.143	5.25	5.25	5.25	5.25
N_c	7	8	8	8	8
$Q_{c,c}$ (W)	0.5968	0.6788	0.7375	0.7745	0.784
COP	0.0192	0.0189	0.018	0.0165	0.015
RS_j=0.2 cm^2 K/W					
I_h (A)	6.5338	7.0084	7.5076	8.0907	9.3278
I_c (A)	7.8165	7.5756	7.6925	7.8118	8.0121
r	6.143	5.25	5.25	5.25	5.25
N_c	7	8	8	8	8
$Q_{c,c}$ (W)	0.6544	0.717	0.782	0.8368	0.8826
COP	0.0219	0.0217	0.0212	0.0201	0.0168
RS_j=2 cm^2 K/W					
I_h (A)	4.4163	5.5156	6.9828	7.9011	9.609
I_c (A)	10.722	10.759	10.866	10.581	11
r	7.333	6.143	5.25	4.556	4.556
N_c	6	7	8	9	9
$Q_{c,c}$ (W)	1.201	1.5826	1.9754	2.1289	2.254
COP	0.0654	0.0631	0.0559	0.0506	0.0393

Table 4
Optimal output variables for A to E Pareto optimal front shown in Fig. 6.

Output variable	Design point				
	A	B	C	D	E
RS_j=0.02 cm^2 K/W					
I_h (A)	7.1558	7.4227	7.7519	8.002	8.5737
I_c (A)	7.1558	7.4227	7.7519	7.6351	8.5737
r	5.25	5.25	5.25	5.25	6.143
N_c	8	8	8	8	7
$Q_{c,c}$ (W)	0.6405	0.6785	0.7127	0.7294	0.7479
COP	0.0191	0.0189	0.0184	0.0178	0.0159
RS_j=0.2 cm^2 K/W					
I_h (A)	7.1681	7.4634	7.7568	8.223	8.7375
I_c (A)	7.1681	7.4634	7.7568	8.223	8.7375
r	6.143	5.25	5.25	5.25	6.143
N_c	8	8	8	8	7
$Q_{c,c}$ (W)	0.7098	0.7563	0.7915	0.825	0.8338
COP	0.0215	0.0209	0.0204	0.0191	0.0172
RS_j=2 cm^2 K/W					
I_h (A)	7.305	7.77	8.285	9.32	10.387
I_c (A)	7.305	7.77	8.285	9.32	10.387
r	3.546	3.546	3.546	4	4.556
N_c	11	11	11	10	9
$Q_{c,c}$ (W)	1.6947	1.868	2.02	2.2258	2.276
COP	0.0506	0.0499	0.0481	0.0426	0.0354

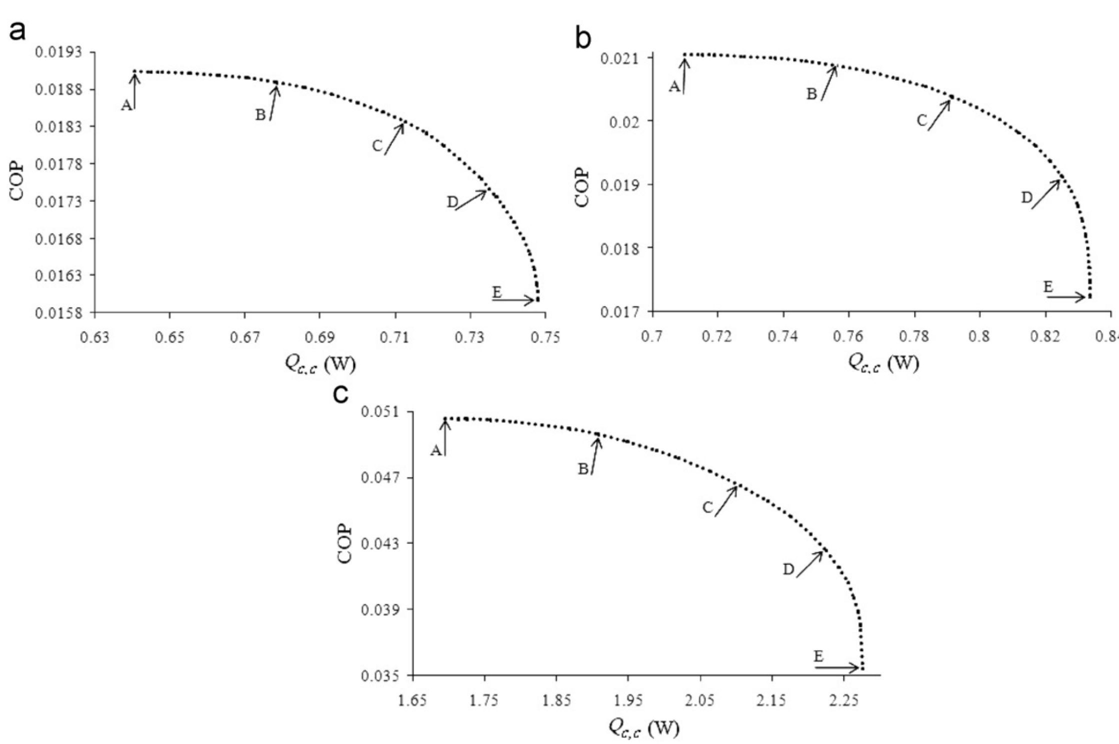

Fig. 5. The distribution of Pareto-optimal point solutions for electrically series connected TEC using the modified TLBO algorithm. (a) RS_j=0.02 cm^2 K/W, (b) RS_j=0.2 cm^2 K/W and (c) RS_j=2 cm^2 K/W.

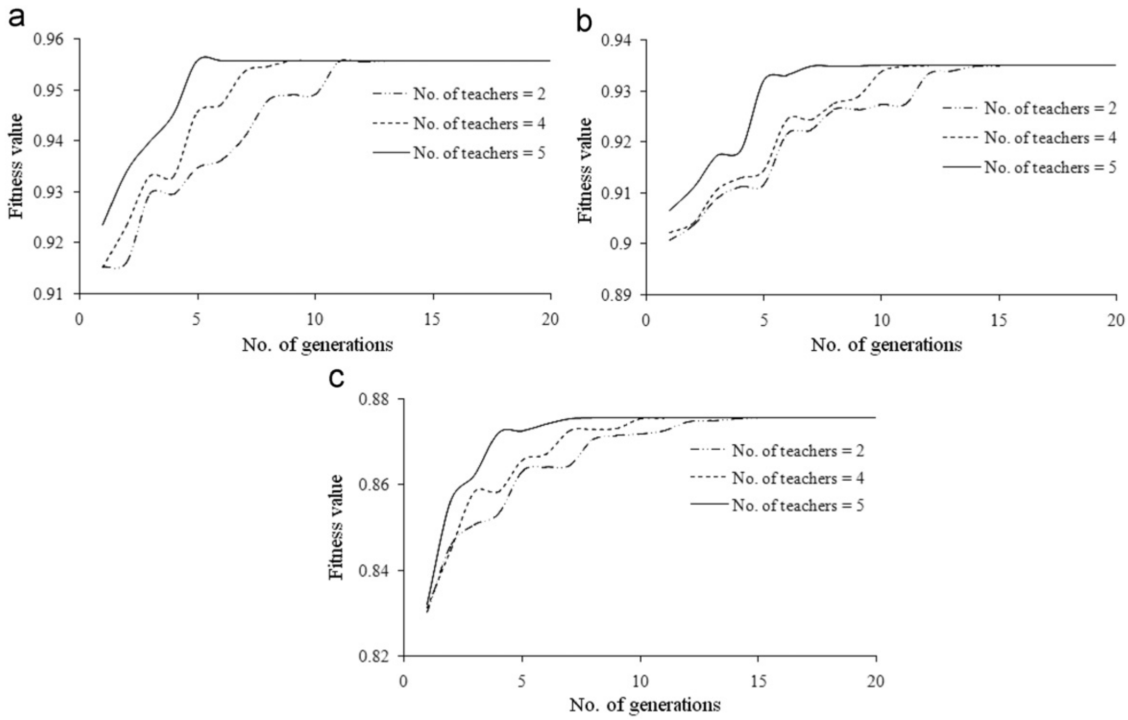

Fig. 6. Effect of number of teachers on the convergence rate of the modified TLBO algorithm for multi-objective consideration (electrically separated TEC).

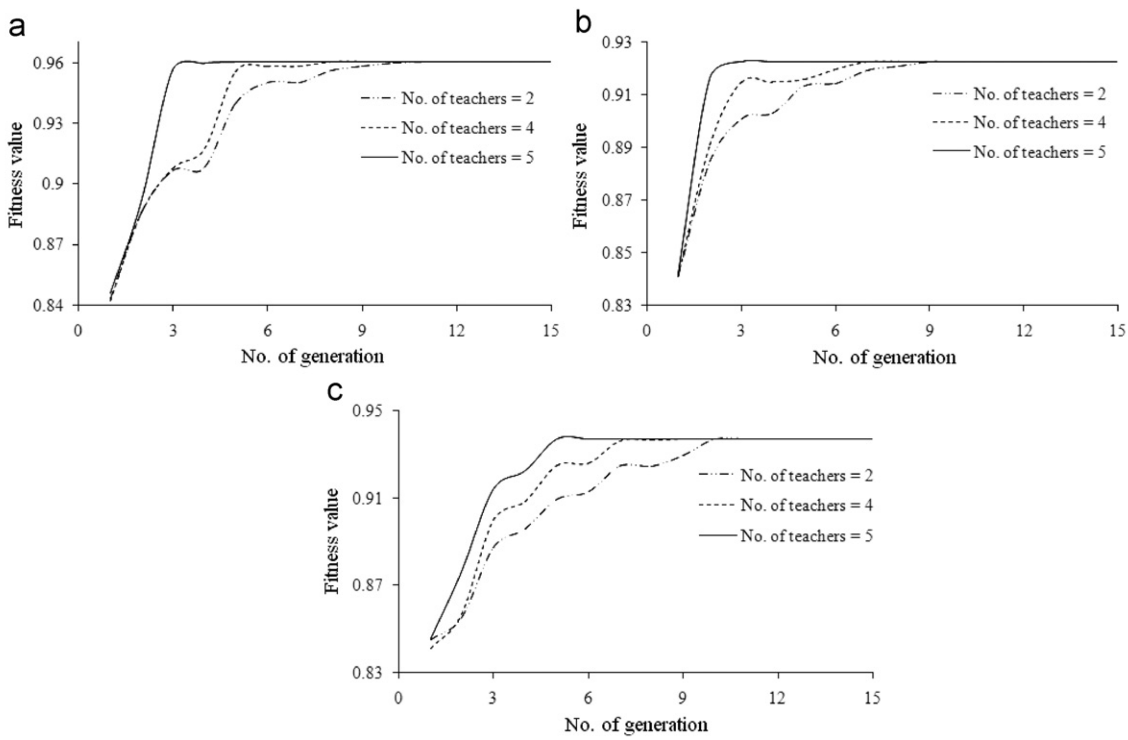

Fig. 7. Effect of number of teachers on the convergence rate of the modified TLBO algorithm for multi-objective consideration (TEC electrically connected in series).

electrically separated. It is observed from Fig. 10 that as the joint resistance increases from 0.2 to 2 cm2 K/W the optimum value of r at which cooling capacity and COP become maximum is reduced. This behavior reflects that the more number of TE modules is required for cold stage as the joint resistance is increased. The effect of I_h, I_c and r on the cooling capacity and COP of the electrically series connected TEC is also very similar to the electrically separated TEC.

To evaluate the performance of modified TLBO algorithm, Sign and Wilcoxon statistical tests are applied to the considered algorithms. The details of these tests are available in the work of Derrac et al. (2011). Multi-objective function of electrically separated TEC with weight factor (w_1) of 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8 and 0.9 are considered for both the tests. All the algorithms are run 50 times with population size of 15 and number of generations of 30.

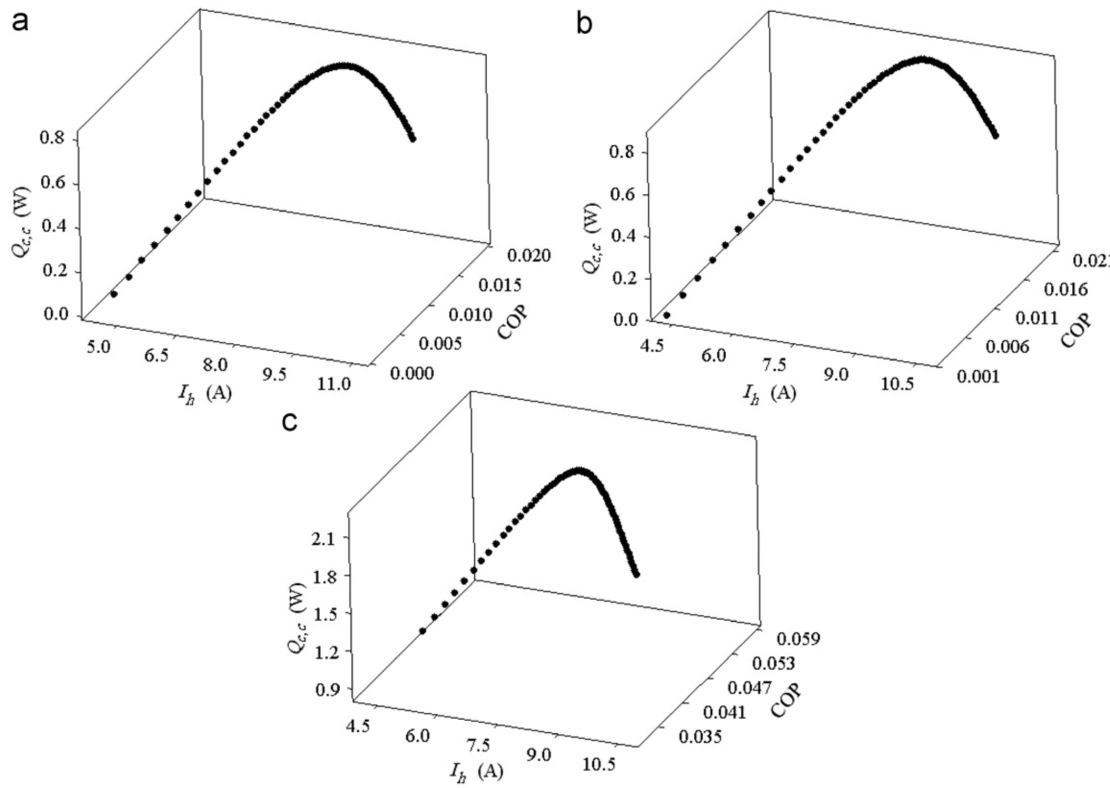

Fig. 8. Sensitivity of the cooling capacity and COP to hot side input current (I_h) for multi-objective consideration (electrically separated TEC). (a) $RS_j=0.02$ cm^2 K/W, (b) $RS_j=0.2$ cm^2 K/W and (c) $RS_j=2$ cm^2 K/W.

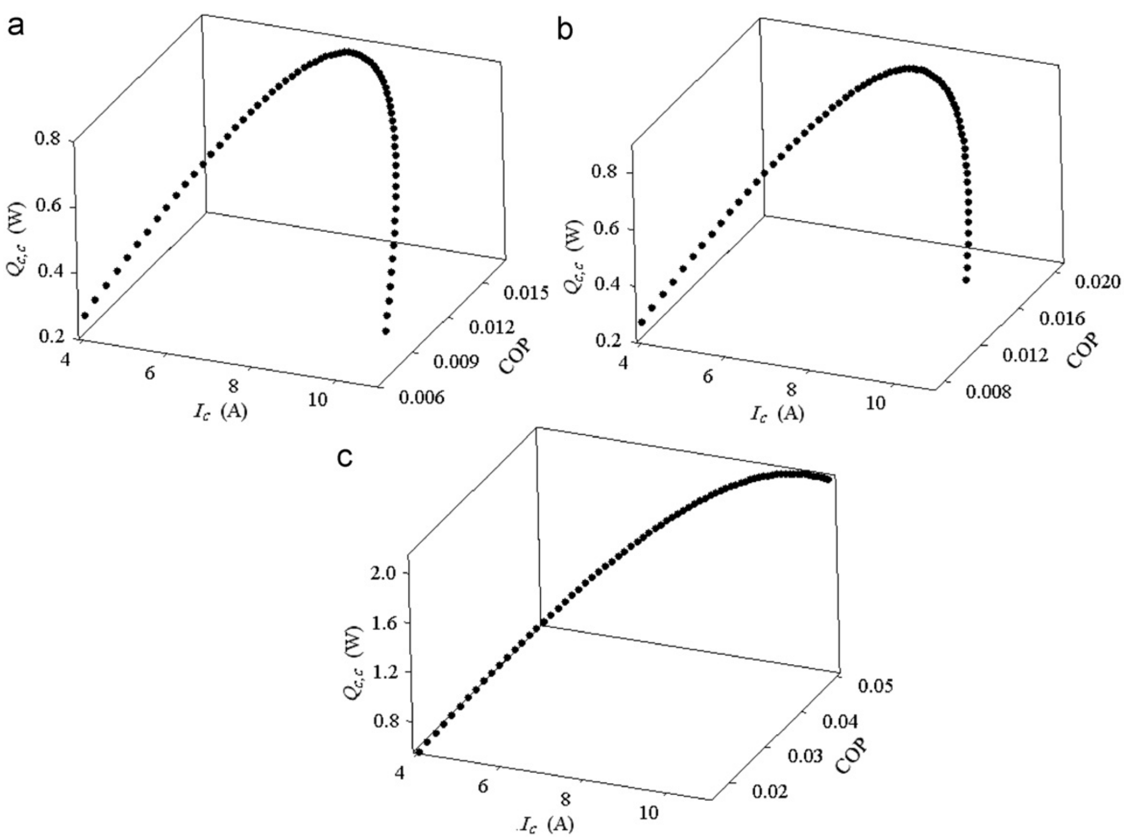

Fig. 9. Sensitivity of the cooling capacity and COP to cold side input current (I_c) for multi-objective consideration (electrically separated TEC). (a) $RS_j=0.02$ cm^2 K/W, (b) $RS_j=0.2$ cm^2 K/W and (c) $RS_j=2$ cm^2 K/W.

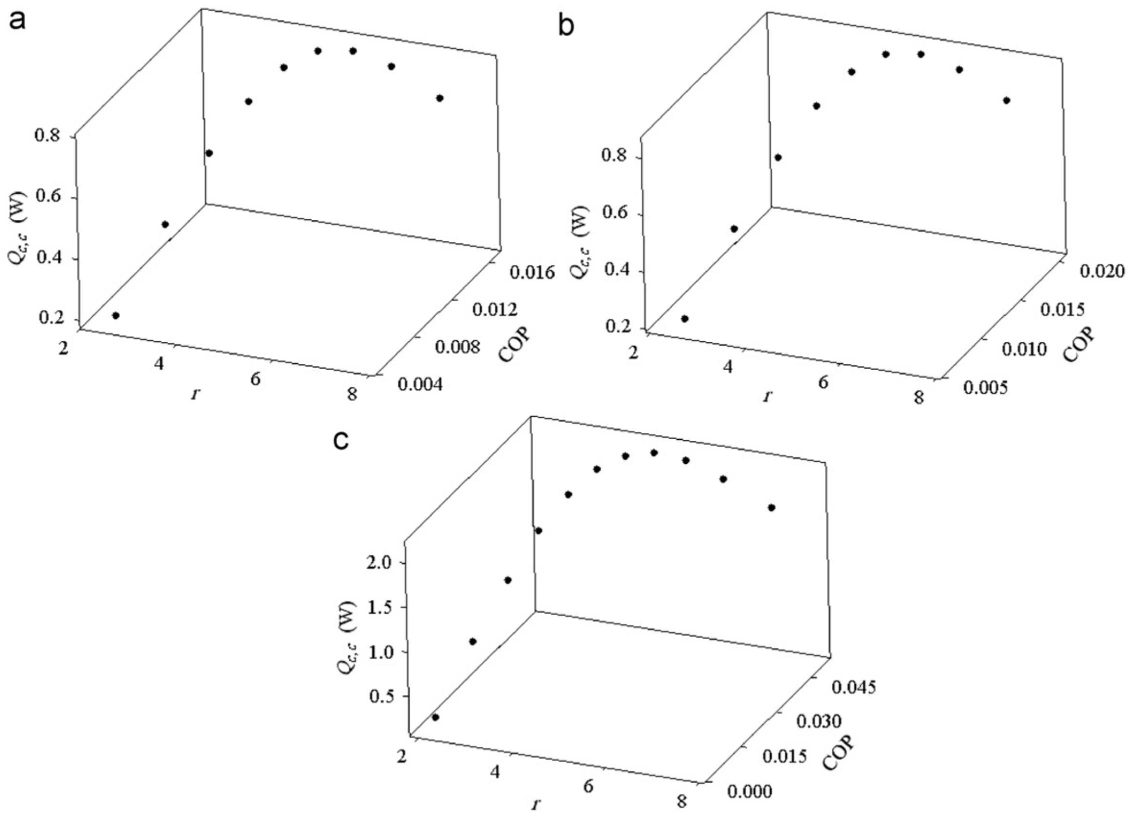

Fig. 10. Sensitivity of the cooling capacity and COP to TE modules for multi-objective consideration (electrically separated TEC). (a) $RS_j = 0.02$ cm^2 K/W, (b) $RS_j = 0.2$ cm^2 K/W and (c) $RS_j = 2$ cm^2 K/W.

Table 5(a)
Result of Sign test.

Modified TLBO	PSO	ABC	TLBO
Wins (+)	9	9	9
Loses (−)	0	0	0
Detected differences	$\alpha = 0.05$	$\alpha = 0.05$	$\alpha = 0.05$

Table 5(b)
Result of Wilcoxon's test.

Comparison	R^+	R^-	p-Value
Modified TLBO versus PSO	43	2	0.00012
Modified TLBO versus ABC	39	6	0.0034
Modified TLBO versus TLBO	34	11	0.0087

In Sign test, performance of the modified TLBO algorithm is evaluated by counting the number of wins achieved either by modified TLBO or by comparison algorithms. Table 5(a) summarizes the results of Sign test. It is observed from Table 5(a) that modified TLBO shows improvement over PSO, ABC and TLBO with a level of significance (α) of 0.05.

In Wilcoxon's test, the difference between the performance scores of two algorithms is ranked according to their absolute value. In the ranking, R^+ indicates the sum of ranks for the problem in which the first algorithm outperformed the second and R^- indicates the sum of ranks for the opposite. After obtaining R^+ and R^-, their associated p-values have been computed. A p-value provides information about whether a statistical test is significant or not, and it also indicates how significant the result is: smaller the p-value, greater the difference between the considered algorithms. Table 5(b) summarizes the results of Wilcoxon's test. It is observed from the results that the modified TLBO method shows improvement over PSO, ABC and TLBO with a level of significance (α) of 0.05.

To identify the computational complexity of the TLBO and modified TLBO algorithms, multi-objective function of electrically separated TEC with weight factor (w_1) of 0.25, 0.5 and 0.75 are considered in the present work. Computational complexity is calculated using the procedure specified by Suganthan et al. (2005). The complexity of TLBO and modified TLBO is calculated as $(T_{2(mean)} - T_1)/T_0$. Where, T_0 is the time to calculate the following:

For $i = 1 : 100$
$x = $ (double) 5.55
$x = x + x$; $x = x/2$; $x = x{*}x$; $x = \mathrm{sqrt}(x)$; $x = \ln(x)$;
$x = \exp(x)$; $y = x/x$;
end

T_1 is the time to calculate multi-objective function for 20 evaluations. It is calculated for each weight factor. T_2 is the mean time for the optimization algorithm to calculate the same function. $T_{2(mean)}$ is the mean time of five execution, again for each weight factor, but now considering the complete computing time of the algorithm for multi-objective function. The complexity of the algorithm is given in Table 6. It is observed from the results that time complexity of the modified TLBO algorithm is higher compared to TLBO algorithm and it is mainly because of the sorting procedure of the modified TLBO algorithm.

8. Conclusion

In the present work, basic TLBO algorithm is modified by introducing the concept of number of teachers, adaptive teaching factor and self-motivated learning. The presented modifications

Table 6
Computational complexity.

	TLBO			Modified TLBO		
	$w_1=0.25$	$w_1=0.50$	$w_1=0.75$	$w_1=0.25$	$w_1=0.50$	$w_1=0.75$
T_0	0.1421	0.1421	0.1421	0.1421	0.1421	0.1421
T_1	89.79	89.63	89.12	93.82	93.11	92.89
$T_{2(mean)}$	191.37	190.41	187.76	199.59	197.64	193.21
Complexity	714.849	709.219	694.159	744.335	735.609	705.982

speed up the convergence rate of the basic TLBO algorithm. The modified TLBO algorithm is applied successfully to the multi-objective optimization of a two stage TEC considering two conflicting objectives: cooling capacity and COP. Two different configurations of TECs, electrically separated and electrically connected in series are investigated for the optimization. Moreover, the contact and spreading resistance of TEC are also considered. The ability of the proposed algorithm is demonstrated by using an example and the performance of the modified TLBO algorithm is compared with the performance of basic TLBO and GA. Improvements in the results are observed using the basic TLBO and modified TLBO algorithms as compared to the GA approach showing the improvement potential of the proposed algorithm for such thermodynamic optimization. The proposed algorithm can be easily customized to suit the optimization of other types of thermal systems involving large number of variables and objectives. These features boost up the applicability of the proposed algorithm for the thermal systems optimization.

Appendix. implementation of modified TLBO for optimization

Step-wise procedure for the implementation of modified TLBO algorithm for one generation is given here.

Step 1: define optimization problem and initialize the optimization parameters.

The following parameters are considered for the cooling capacity maximization.

Population size $(P_n)=15$ Number of generations $(G_n)=1$

Number of design variables $(D_n)=3$ No. of teachers$=2$

Define optimization problem as

Maximize $f(X)=Q_{c,c}(X)$, $X=[I_h, I_c, r]$.

Step 2: initialize population.

Generate random population. Short the population according to their respective objective function value. This population is expressed as

$$=\begin{bmatrix} 8.872 & 8.716 & 4.556 \\ 7.972 & 8.948 & 5.25 \\ 8.140 & 9.536 & 6.143 \\ 9.217 & 9.062 & 2.84 \\ 6.730 & 8.521 & 6.143 \\ 8.061 & 6.123 & 3.545 \\ 8.831 & 6.697 & 6.143 \\ 6.119 & 7.411 & 5.25 \\ 5.037 & 8.781 & 6.143 \\ 5.389 & 7.462 & 5.25 \\ 6.983 & 5.950 & 5.25 \\ 10.073 & 5.126 & 2.33 \\ 7.666 & 10.117 & 2.33 \\ 4.942 & 10.023 & 3.167 \\ 4.877 & 5.702 & 4.556 \end{bmatrix}$$

$$\text{and corresponding value of objective function} = \begin{bmatrix} 2.05 \\ 1.953 \\ 1.942 \\ 1.901 \\ 1.62 \\ 1.459 \\ 1.424 \\ 1.365 \\ 1.262 \\ 1.182 \\ 1.146 \\ 1.038 \\ 0.844 \\ 0.693 \\ 0.564 \end{bmatrix}$$

Step 3: selection of teachers.

Based on the initial population the best solution will act as a chief teacher for that iteration

$$X_{1,teacher}=f(X)_{min}$$
$$=[8.872\ \ 8.716\ \ 4.556] \text{ and corresponding value of objective function}=[2.05]$$

Based on the chief teacher the second teacher is obtained by using the mathematical expression explained in Section 4.3

$$X_{2,teacher}=f(X)$$
$$=[6.983\ \ 5.95\ \ 5.25] \text{ with the objective function value}=[1.146]$$

Step 4: assign the learners to the teachers according to their objective function value.

Group 1

$$=\begin{bmatrix} 8.872 & 8.716 & 4.556 \\ 7.972 & 8.948 & 5.25 \\ 8.140 & 9.536 & 6.143 \\ 9.217 & 9.062 & 2.84 \\ 6.730 & 8.521 & 6.143 \\ 8.061 & 6.123 & 3.545 \\ 8.831 & 6.697 & 6.143 \\ 6.119 & 7.411 & 5.25 \\ 5.037 & 8.781 & 6.143 \\ 5.389 & 7.462 & 5.25 \end{bmatrix}$$

$$\text{and corresponding value of objective function} = \begin{bmatrix} 2.05 \\ 1.953 \\ 1.942 \\ 1.901 \\ 1.62 \\ 1.459 \\ 1.424 \\ 1.365 \\ 1.262 \\ 1.182 \end{bmatrix}$$

$$f(X)_{average}=1.6158$$

Group 2

$$=\begin{bmatrix} 6.983 & 5.950 & 5.25 \\ 10.073 & 5.126 & 2.33 \\ 7.666 & 10.117 & 2.33 \\ 4.942 & 10.023 & 3.167 \\ 4.877 & 5.702 & 4.556 \end{bmatrix}$$

$$\text{and corresponding value of objective function} = \begin{bmatrix} 1.146 \\ 1.038 \\ 0.844 \\ 0.693 \\ 0.564 \end{bmatrix}$$

$f(X)_{average} = 0.857$

Step 5: teacher phase.

For each group calculate the mean of the learners column wise, which will give the mean for the particular subject as

$$M_{s,D} = [m_1, m_2, \ldots, m_D]$$
$$= [7.437 \;\; 8.126 \;\; 5.1] \text{ for group 1 and}$$
$$= [6.908 \;\; 7.384 \;\; 3.527] \text{ for group 2}$$

Each teacher try to shift the mean of their group from $M_{s,D}$ towards $X_{s,teacher}$, which will act as a new mean for the iteration. So

$$M\_new_{s,D} = X_{s,D}$$

And difference between two means is expressed as

$$\text{Difference\_Mean}_{s,D} = r(M\_new_{s,D} - T_{F_s} M_{s,D})$$

where T_F is adaptive teaching factor obtained by using the mathematical expression explained in Section 4.2

$$= [0.838 \;\; 0.932 \;\; 1.119] \text{ for group 1 and}$$
$$= [0.989 \;\; 1.241 \;\; 0.672] \text{ for group 2.}$$

The obtained difference is added to the current solution to update its values as

$$X_{new,D} = X_{old,D} + \text{Difference}_{s,D}$$

Group 1

$$X_{new} = \begin{bmatrix} 9.466 & 9.784 & 4 \\ 8.565 & 10.016 & 4.556 \\ 8.734 & 10.604 & 5.25 \\ 9.811 & 10.13 & 2.235 \\ 7.324 & 9.589 & 5.538 \\ 8.655 & 7.191 & 2.33 \\ 9.424 & 7.766 & 5.25 \\ 6.713 & 8.479 & 4.556 \\ 5.631 & 9.849 & 5.25 \\ 5.983 & 8.529 & 4.556 \end{bmatrix}$$

$$\text{and corresponding value of objective function} = \begin{bmatrix} 2.201 \\ 2.176 \\ 2.184 \\ 1.194 \\ 1.949 \\ 1.505 \\ 1.804 \\ 1.736 \\ 1.582 \\ 1.556 \end{bmatrix}$$

Group 2

$$X_{new} = \begin{bmatrix} 7.117 & 5.391 & 6.143 \\ 10.208 & 4.567 & 2.846 \\ 7.800 & 9.558 & 2.846 \\ 5.076 & 9.464 & 4.000 \\ 5.011 & 5.143 & 5.250 \end{bmatrix}$$

$$\text{and corresponding value of objective function} = \begin{bmatrix} 0.918 \\ 0.861 \\ 1.671 \\ 1.231 \\ 0.434 \end{bmatrix}$$

Accept X_{new} if it gives better function value.

Group 1

$$X_{new} = \begin{bmatrix} 9.466 & 9.784 & 4 \\ 8.565 & 10.016 & 4.556 \\ 8.734 & 10.604 & 5.25 \\ 9.217 & 9.062 & 2.84 \\ 7.324 & 9.589 & 5.538 \\ 8.655 & 7.191 & 2.33 \\ 9.424 & 7.766 & 5.25 \\ 6.713 & 8.479 & 4.556 \\ 5.631 & 9.849 & 5.25 \\ 5.983 & 8.529 & 4.556 \end{bmatrix}$$

$$\text{and corresponding objective function value} = \begin{bmatrix} 2.201 \\ 2.176 \\ 2.184 \\ 1.901 \\ 1.949 \\ 1.505 \\ 1.804 \\ 1.736 \\ 1.582 \\ 1.556 \end{bmatrix}$$

$f(X)_{average} = 1.8594$

Group 2

$$X_{new} = \begin{bmatrix} 6.983 & 5.950 & 5.25 \\ 10.073 & 5.126 & 2.33 \\ 7.800 & 9.558 & 2.846 \\ 5.076 & 9.464 & 4.000 \\ 4.877 & 5.702 & 4.556 \end{bmatrix}$$

$$\text{and corresponding objective function value} = \begin{bmatrix} 1.146 \\ 1.038 \\ 1.671 \\ 1.231 \\ 0.564 \end{bmatrix}$$

$f(X)_{average} = 1.13$

Step 6: learner phase.

Learners increase their knowledge with the help of their mutual interaction. Mathematical expression is explained under Section 4.3. The new solution obtained after learner phase is accepted if it gives better results than the teacher phase. Obtain X_{new} after the student phase,

Group 1

$$X_{new}=\begin{bmatrix} 9.360 & 10.964 & 5.250 \\ 8.565 & 10.016 & 4.556 \\ 8.734 & 10.604 & 5.25 \\ 9.217 & 9.062 & 2.84 \\ 7.324 & 9.589 & 5.538 \\ 9.280 & 8.294 & 5.250 \\ 9.424 & 7.766 & 5.25 \\ 10.126 & 10.988 & 6.143 \\ 6.290 & 9.453 & 6.143 \\ 9.462 & 8.817 & 4.556 \end{bmatrix}$$

$$\text{and corresponding objective function value}=\begin{bmatrix} 2.224 \\ 2.176 \\ 2.184 \\ 1.901 \\ 1.949 \\ 1.912 \\ 1.804 \\ 2.121 \\ 1.668 \\ 2.08 \end{bmatrix}$$

$f(X)_{average}=2.0019$

Group 2

$$X_{new}=\begin{bmatrix} 8.953 & 9.933 & 5.250 \\ 5.167 & 9.438 & 5.250 \\ 7.800 & 9.558 & 2.846 \\ 10.273 & 6.318 & 5.250 \\ 4.877 & 5.702 & 4.556 \end{bmatrix}$$

$$\text{and corresponding objective function value}=\begin{bmatrix} 2.141 \\ 1.398 \\ 1.671 \\ 1.395 \\ 0.564 \end{bmatrix}$$

$f(X)_{average}=1.4338$

Step 7: self-motivated learning.

Learners of each group try to improvise their knowledge by self-learning. Mathematical expression is explained under Section 4.3. The new solution obtained after this phase is accepted if it gives better results than the learner phase. Obtain X_{new} after the self-motivated learning phase,

Group 1

$$X_{new}=\begin{bmatrix} 9.660 & 10.980 & 5.250 \\ 8.565 & 10.016 & 4.556 \\ 8.734 & 10.604 & 5.25 \\ 8.928 & 9.294 & 5.250 \\ 7.324 & 9.589 & 5.538 \\ 10.360 & 9.350 & 6.143 \\ 9.287 & 9.307 & 5.250 \\ 10.126 & 10.988 & 6.143 \\ 9.356 & 9.318 & 6.143 \\ 10.158 & 9.970 & 5.250 \end{bmatrix}$$

$$\text{and corresponding objective function value}=\begin{bmatrix} 2.225 \\ 2.176 \\ 2.184 \\ 2.07 \\ 1.949 \\ 1.936 \\ 2.08 \\ 2.121 \\ 1.96 \\ 2.137 \end{bmatrix}$$

$f(X)_{average}=2.0838$

Group 2

$$X_{new}=\begin{bmatrix} 8.953 & 9.933 & 5.250 \\ 9.421 & 8.924 & 6.143 \\ 7.800 & 9.558 & 2.846 \\ 10.273 & 6.318 & 5.250 \\ 9.234 & 9.475 & 6.143 \end{bmatrix}$$

$$\text{and corresponding objective function value}=\begin{bmatrix} 2.141 \\ 1.9 \\ 1.671 \\ 1.395 \\ 1.981 \end{bmatrix}$$

$f(X)_{average}=1.8176$

Step 8: combine all group.

Step 9: termination criterion.

Stop if maximum generation number is achieved; otherwise repeat from step 3.

It is observed from the above results that the average ($f(X)_{average}$) and the best value of the objective function increase as algorithm proceeds from the teacher phase to learner phase and learner phase to self-motivated learning phase in the same generation. In the same way the objective function value is also increased with the number of generations.

References

Abramzon, B., 2007. Numerical optimization of the thermoelectric cooling devices. J. Electron. Packag. 129 (3), 339–347.

Chen, J., Zhou, Y., Wang, H., Wang, J.T., 2002. Comparison of the optimal performance of single- and two-stage thermoelectric refrigeration systems. Appl. Energy 73, 285–298.

Chen, L., Li, J., Sun, F., Wu, C., 2005. Effect of heat transfer on the performance of two-stage semiconductor thermoelectric refrigerators. J. Appl. Phys. 98 (3), 1–7.

Chen, L., Li, J., Sun, F., Wu, C., 2008. Performance optimization for a two-stage thermoelectric heat-pump with internal and external irreversibilities. Appl. Energy 85 (7), 641–649.

Chen, X., Lin, B., Chen, J., 2006. The parametric optimum design of a new combined system of semiconductor thermoelectric devices. Appl. Energy 83, 681–686.

Chen, L., Meng, F., Sun, F., 2009. A novel configuration and performance for a two-stage thermoelectric heat pump system driven by a two-stage thermoelectric generator. Proc. Inst. Mech. Eng. A—J. Power 223 (4), 329–339.

Cheng, Y.H., Lin, W.K., 2005. Geometric optimization of thermoelectric coolers in a confined volume using genetic algorithms. Appl. Therm. Eng. 25, 2983–2997.

Cheng, Y.H., Shih, C., 2006. Maximizing the cooling capacity and COP of two-stage thermoelectric coolers through genetic algorithm. Appl. Therm. Eng. 26, 937–947.

Derrac, J., Garcia, S., Molina, D., Herrera, F., 2011. A Practical tutorial on the use of nonparametric statistical tests as methodology for comparing evolutionary intelligence algorithms. Swarm Evol. Comput. 1, 3–18.

Lai, H., Pan, Y., Chen, J., 2004. Optimum design on the performance parameters of a two-stage combined semiconductor thermoelectric heat pump. Semicond. Sci. Technol. 19, 17–22.

Lee, S., Song, S., Au, V., Moran, K.P., 1995. Constriction/spreading resistance model for electronic packaging. In: Proceedings of ASME/JSME Thermal Engineering Joint Conference, vol. 4. Maui, Hawaii, pp. 199-206.

Meng, F., Chen, L., Sun, F., 2009. Performance optimization for two-stage thermoelectric refrigerator system driven by two-stage thermoelectric generator. Cryogenics 49 (2), 57–65.

Pan, Y., Lin, B., Chen, J., 2007. Performance analysis and parametric optimal design of an irreversible multi-couple thermoelectric refrigerator under various operating conditions. Appl. Energy 84, 882–892.
Rao, R.V., Savsani, V.J., Vakharia, D.P., 2011. Teaching–learning-based optimization: a novel method for constrained mechanical design optimization problems. Comput. Aided Des. 43 (3), 303–315.
Rowe, D.M., 1996. CRC handbook of thermoelectric. CRC Press, London.
Suganthan, P.N., Hansen, N., Liang, J.J., Deb, K., Chen, A., Auger, Y.P., Tiwari, S., 2005. Problem Definitions and Evaluation Criteria for the CEC 2005 Special Session on Real-Parameter Optimization. Technical Report. Nanyang Technological University, Singapore. 〈http://www.ntu.edu.sg/home/EPNSugan〉.
Xuan, X.C., 2002. Analyses of the performance and polar characteristics of two-stage thermoelectric coolers. Semicond. Sci. Technol. 17, 414–420.
Xuan, X.C., Ng, K.C., Yap, C., Chua, H.T., 2002a. Optimization of two stage thermoelectric coolers with two design configurations. Energy Convers. Manage. 43, 2041–2052.
Xuan, X.C., Ng, K.C., Yap, C., Chua, H.T., 2002b. The maximum temperature difference and polar characteristic of two-stage thermoelectric coolers. Cryogenics 42, 273–278.
Yu, J., Zhao, H., Xie, K., 2007. Analysis of optimum configuration of two-stage thermoelectric modules. Cryogenics 47 (2), 89–93.